NEW ASPECTS OF THE PROBLEM OF THE SOURCE OF THE KERR SPINNING PARTICLE

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Essay written for the Gravity Research Foundation 2010 Awards for Essays on Gravitation. (March 31, 2010)

Abstract

We consider development of the models of the source of the Kerr-Newman (KN) solution and new aspects related with the obtained recently field model based on a domain wall bubble with superconducting interior. The internal Higgs field regularizes the KN solution, expelling electromagnetic field from interior to the boundary of bubble. The KN source forms a gravitating soliton, interior of which is similar to oscillating solitons (Q-balls, oscillons), while exterior is consistent with the KN solution. We obtain that a closed Wilson loop appears on the edge of the bubble, resulting in quantization of angular momentum of the regularized solutions. A new holographic interpretation of the mysterious twosheetedness of the Kerr geometry is given.

There are many evidences that black holes (BH) are akin to elementary particles [1]. Carter obtained that the KN BH solution has $g = 2$ as that of the Dirac electron, [2], and there followed a series of the works on the problem of the source of KN spinning particle, and on the models of the KN electron consistent with gravity [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Since spin of electron is very high, the horizons of KN solution disappear, opening the naked singular ring which should be replaced by a regular matter source. The regularized BH solutions may be considered as gravitating solitons [13, 14], the nonperturbative field solutions of the electroweak sector of standard model [14, 15] which may realize important bridge between quantum theory and gravity.

Regularization of the KN solution represents a very old and hard problem related with specific *twosheeted structure* of the Kerr geometry [9, 10, 16]. The Kerr-Schild (KS) form of metric is

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H k_\mu k_\nu, \quad H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}, \tag{1}$$

and electromagnetic vector potential is

$$A^\mu_{KN} = Re \frac{e}{r + ia\cos\theta} k^\mu, \tag{2}$$

where $k^\mu(x^\mu)$ is the null vector field tangent to the Kerr principal null congruence (PNC) [2], $k_\mu dx^\mu = dr - dt - a\sin^2\theta d\phi_K$, and $\eta^{\mu\nu}$ is the auxiliary Minkowski metric with Cartesian coordinates $x^\mu = (t, x, y, z)$, related with the Kerr oblate spheroidal coordinates r, θ, ϕ_K as follows

$$x + iy = (r + ia)\exp\{i\phi_K\}\sin\theta, \;\; z = r\cos\theta, \tag{3}$$

where $a = J/m$ is radius of the Kerr singular ring, a branch line of the KN spacetime.

The coordinate r covers the Kerr space-time twice, for $r > 0$ and for $r < 0$, forming the 'positive' and 'negative' sheets connected analytically via disk $r = 0, \cos\theta \le 1$, see Fig.1. The Kerr congruence $k_\mu(x)$ covers the spacetime twice too: in the form of ingoing rays $k^{\mu(-)}$ falling on the disk $r = 0$, and as outgoing rays $k^{\mu(+)}$, for $r > 0$, which leads to different metrics on the in- and out- sheets of the KN solution.

There are different models of the KN source, and the we sketch here some typical ones.

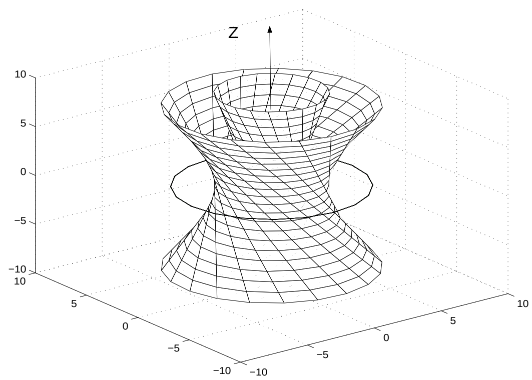

Figure 1: The Kerr singular ring and projection of the Kerr congruence on auxiliary Minkowski background.

(1) Israel [3] (1968) truncated negative KN sheet, replacing it by the *rotating disk*, $r = 0$, spanned by the Kerr singular ring of the Compton radius $a = \hbar/2m$.

(2) In the suggested in [4] (1974) model of *"microgeon with spin"*, the Kerr singular ring was considered as a waveguide for electromagnetic traveling waves generating the spin and mass of the KN solution. Singular ring was interpreted as a *closed 'Alice' string* opening a gate to negative sheet of spacetime [8, 9, 10] (1995,2004,2008).

(3) López, [6] (1984), generalized the Israel model by introducing the ellipsoidal *bubble source* covering the Kerr ring. The external KN solution matches with flat interior along ellipsoidal boundary $r = r_0 = e^2/2m$, forming an oblate rigidly rotating bubble of the Compton size, with the thickness $2r_0 = e^2/m$, equal to the classical size of electron.

(4) The *field and baglike models*: [17] (1974), [18] (1989), [19] (2000), [20] (2002), [11] (2005), [12] (2010) which are regular and similar to gravitating solitons.

The long-term development of the models of KN source resulted in the obtained recently field generalization of the López model, [12], based on a *domain wall bubble* interpolating between the external KN solution and internal superconducting pseudo-vacuum state.

Gravitational sector of the model is described by the metric (1) with the suggested in [17] function $H = f(r)/(r^2 + a^2 \cos^2 \theta)$, which can describe the rotating metrics of different types and match them smoothly, [17, 19, 20].[1] In particular, the external KN metric, $f(r) = mr - e^2/2$, matches with flat interior, $f = 0$, along the ellipsoidal surface $r = r_0 = e^2/2m$.

Electromagnetic – Higgs sector is described by Higgs Lagrangian, [21],

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \mathcal{D}_\mu \Phi \bar{\mathcal{D}}^\mu \bar{\Phi} + V, \tag{4}$$

where $\mathcal{D}_\mu = \nabla_\mu + ieA_\mu$; $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$; $\Phi = \Phi_0 \exp\{i\chi\}$, leading to

$$\Box A_\mu = I_\mu = e|\Phi|^2 (\chi,_\mu + eA_\mu). \tag{5}$$

Potential V provides the phase transition from external KN solution, where $\Phi = 0$, to superconducting internal state with $|\Phi| = \Phi_0 > 0$.

The bizarre Kerr coordinate ϕ_K, (3), is inconsistent with the Higgs angular coordinate ϕ. After coordinate transformations $\phi_K \to \phi$, the potential (2) takes the form

$$A_\mu dx^\mu|_r = \frac{-er}{r^2 + a^2 \cos^2 \theta} [dt + a \sin^2 \theta d\phi] + \frac{2er dr}{(r^2 + a^2)}. \tag{6}$$

It increases, approaching the boundary of bubble $r = r_0 = e^2/2m$, and in the equatorial plane, $\cos \theta = 0$, it reaches the magnitude

$$A_\mu^{(edge)} dx^\mu = -\frac{2m}{e} [dt + a d\phi] + \frac{2er_0 dr}{(r_0^2 + a^2)}. \tag{7}$$

The directions $A_\mu^{(edge)}$ in the equatorial plane are tangent to the Kerr singular ring and form a closed loop at the edge. The Wilson loop integral

$$S^{(edge)} = \oint_{(edge)} A_\mu(x) dx^\mu = \oint e A_\phi^{(edge)} d\phi = -4\pi m a = -4\pi J \tag{8}$$

turns out to be proportional to the KN angular momentum.

The Higgs field $\Phi(x) = \Phi_0 e^{i\chi(x)}$ expels the electromagnetic field and current from the bulk of the superconducting bubble, and we should set $I_\mu = 0$ for $r < r_0$. It gives the internal solution

$$\chi,_\mu = -eA_\mu^{(in)}, \tag{9}$$

[1] The Kerr geometry is foliated into rigidly rotating ellipsoidal layers $r = const.$ with angular velocities $\Omega(r) = \frac{a}{a^2+r^2}$.

as a full differential, and the second equation $\Box A_\mu^{(in)} = 0$ is satisfied automatically. Taking the Higgs phase in general form $\chi = \omega t + n\phi + \chi_1(r)$, one obtains from (10) the internal solution

$$A_0^{(in)} = -\frac{\omega}{e}; \; A_\phi^{(in)} = -\frac{n}{e}; \; A_r^{(in)} = \chi_1'(r)/e. \tag{10}$$

Matching the edge field (7) with internal one, we obtain

$$\omega = 2m; \; J = ma = n/2; \; \chi_1(r) = -\ln(r^2 + a^2), \tag{11}$$

and therefore

$$\Phi(x) = \Phi_0 \exp\{i\chi\} = \Phi_0 \exp\{i2mt - i\ln(r^2 + a^2) + in\phi\}. \tag{12}$$

Two important consequences follow from (11):

i)The Higgs field forms a coherent vacuum state oscillating with the frequency $\omega = 2m$, similar to the soliton models of the spinning Q-balls [22, 23] and bosonic stars [24].

ii) Angular momentum of the regular bubble source of the KN solution is quantized, $J = ma = n/2, \; n = 1, 2, 3, ...$

The electromagnetic field and currents in a superconductor have a 'penetration depth' $\delta \sim \frac{1}{e|\Phi|} = 1/m_v$, [21]. In our case it forms a thin surface layer, $r_0 - \delta < r < r_0$, in which the potential A_μ differs from the obtained solution (10). Its deviation, $A_\mu^{(\delta)} = A_\mu - A_\mu^{(in)}$ obeys the massive equation

$$\Box A_\mu^{(\delta)} = m_v^2 A_\mu^{(\delta)} \; , \tag{13}$$

which shows that a massive vector meson with mass $m_v = e|\Phi|$ resides at the KN bubble and generates the circular current $I_\mu = e m_v^2 A_\mu^{(\delta)}$ concentrating at the edge of bubble close to Wilson loop. There may also be a spectrum of such solutions, which supports the stringy version (2) of the KN source.

Although in the considered model the Kerr singular ring is removed and the internal space is flat, the used oblate coordinate system still contains the harmless ringlike coordinate singularity, and the KN twosheetedness has been survived. The inner superconducting state may be extended analytically to negative sheet, $r < 0$, forming a flat superconducting pseudo-vacuum state, having the zero total energy density. We arrive at a *holographic interpretation* of the KN twosheetedness [25] which turns out to be necessary for quantum treatment. The negative sheet is considered as an in-vacuum space, separated

from the physical out-sheet by the holographically dual boundary of the bubble. The necessity of such separation was suggested in particular by Gibbons, [27], who separated the curved spacetime $\mathcal{M}$ into two time-ordered regions $\mathcal{M}_-$ and $\mathcal{M}_+$ associated with ingoing and outgoing vacuum states $|0_- >$ and $|0_+ >$. Similar prequantum spacetime was introduced for black holes by ‘t Hooft et.al. in [28]: the two sheets of the KN space correspond to the ‘t Hooft holographic correspondence, in which the source forms a membrane, holographically dual to the bulk $\mathcal{M} = \mathcal{M}_- \bigcup \mathcal{M}_+$, [25]. The mysterious problem of twosheetedness of the KN space-time turns into its advantage related with a holographic Kerr-Schild structure [25] adapted for quantum treatment [27].

The obtained solitonlike source of the KN solution represents a bubble filled by coherently oscillating Higgs field, the typical feature of the other ‘oscillon’ or ‘breather’ soliton solutions. For parameters of electron $a = \hbar/2m \sim 10^{22}$, and the bubble forms a strongly oblated, rotating disk of the Compton radius, which corresponds to the size of electron dressed by virtual photons. We arrive at the conclusion that the obtained inner coherent structure of the Compton region, as well as the adjoined Wilson loop and circular current[2], should apparently be considered as integral parts of the consistent with gravity solitonlike electron structure.

References

[1] G. ’t Hooft, Nucl. Phys. B **335**, 138 (1990); C. F. E. Holzhey and F. Wilczek, Nucl. Phys. B **380**, 447 (1992); A. Sen, Mod. Phys. Lett. A **10** 2081 (1995);

[2] G.C. Debney, R.P. Kerr, A.Schild, J. Math. Phys. **10**(1969) 1842.

[3] W. Israel, Phys.Rev. D **2**, 641 (1970).

[4] A.Ya. Burinskii, *Sov. Phys. JETP,* **39**, 193 (1974); *Russian Phys. J.* **17**, 1068 (1974).

[5] V. Hamity, Phys.Lett. A **56**, 77 (1976).

[6] C.A. López, *Phys. Rev.* **D 30** 313 (1984).

[2]Note, that circular currents of the Compton size were experimentally confirmed long ago in the low energy absorbtion of the γ-rays in aluminium [29].

[7] A. Burinskii Phys.Rev. D **70**, 086006 (2004); hep-th/0406063.

[8] A. Burinskii, Grav. Cosmol. **14** (2008) 109, arXiv:hep-th/0507109.

[9] A. Burinskii, Grav.Cosmol.**10**, (2004) 50; hep-th/0403212, hep-th/0507109.

[10] A. Burinskii, Phys.Rev. D **52**, 5826 (1995).

[11] I. Dymnikova, Phys. Lett. **B639**, 368 (2006).

[12] A. Burinskii, arXiv: 1003.2928[hep-th]; arXiv: 0910.5388[hep-th].

[13] B.Kleihaus, J.Kunz, Ya. Shnir, Phys.Rev. **D70**, 065010 (2004); J.Kunz, U.Neemann, Ya.Shnir Phys.Rev.**D75**, 125008 (2007).

[14] N.S.Manton, Phys. Rev.**D 28**, 2019 (1983);

E. Radu and M.Volkov, Phys.Rept.**468** 101-151 (2008).

[15] R.F. Dashen, B.Hasslacher and A. Neveu, Phys. Rev. **D 10**, 4138 (1974);

[16] A. Burinskii, First Award of GRF 2009, Gen. Rel. Grav. **41** 2281, arXiv: 0903.3162.

[17] M. Gürses and F. Gürsey, J. Math. Phys. **16**, 2385 (1975).

[18] A. Burinskii, Phys.Lett.B **216**, 123 (1989).

[19] A. Burinskii, Grav. Cosmol. **8**, 261 (2002), arXiv:hep-th/0110011; J.Phys. A: Math.Gen. **39**, 6209 (2006); Int.J.Mod.Phys. A **17**, 920 (2002).

[20] A. Burinskii, E.Elizalde, S. R. Hildebrandt and G. Magli, Phys.Rev. D **65**, 064039 (2002), arXiv:gr-qc/0109085.

[21] H.B. Nielsen and P. Olesen, Nucl. Phys. B **61**, 45 (1973).

[22] M.Volkov and E. Wöhnert, Phys.Rev. D **66**, 085003 (2002).

[23] N.Graham, Phys.Rev.Lett **98**, 101801 (2007).

[24] F.E.Schunck and E.W.Mielke, in *Relativity and Scientific Computing,* edited by F.Hehl at.al., Springer, Berlin, 1966, pp.138-151.

Sh.Yoshida and Yo.Eriguchi, Phys.Rev. **D56**, 762(1997);

[25] A. Burinskii, Theor. Math. Phys. 2010 (to appear); arXiv:1001.0332[gr-qc] .

[26] A. Burinskii, Grav. Cosmol. **11**, 301 (2005), hep-th/0506006; IJGMMP, **4**, n.3, 437 (2007).

[27] G. W. Gibbons, Comm. Math. Phys. **45** 191 (1975);

[28] C.R. Stephens, G. 't Hooft and B.F. Whiting, Class. Quant. Grav. **11** 621 (1994).

[29] A.H. Compton, Phys.Rev. **14**, 247 (1919).